

Some connections between BCK -algebras and n -ary block codes

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Abstract. In the last time some papers were devoted to the study of the connections between binary block codes and BCK -algebras. In this paper, we try to generalize these results to n -ary block codes, providing an algorithm which allows us to construct a BCK -algebra from a given n -ary block code.

Keywords: BCK -algebras; n -ary block codes.

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0. Introduction

Y. Imai and K. Iseki introduced BCK -algebras in 1966, through the paper [Im, Is; 66], as a generalization of the concept of set-theoretic difference and propositional calculi. This class of BCK -algebras is a proper subclass of the class of BCI -algebras and has many applications to various domains of mathematics.

One of the recent applications of BCK -algebras was given in the Coding Theory. In the paper [Ju, So; 11], the authors constructed a finite binary block-codes associated to a finite BCK -algebra. In [Fl; 15], the author proved that, in some circumstances, the converse of the above statement is also true and in the paper [B, F; 15] the authors proved that binary block codes are an important tool in providing orders with which we can build algebras with some asked properties. For other details regarding BCK -algebras, the reader is referred to [Is, Ta; 78].

In general, the alphabet on which are defined block codes are not binary. It is used an alphabet with n elements, $n \geq 2$, identified usually with the set $A_n = \{0, 1, 2, \dots, n-1\}$. These codes are called n -ary block codes. In the present paper, we will generalize this construction of binary block codes to n -ary block codes. For this purpose, we will prove that to each n -ary block code V we can associate a BCK -algebra X such that the n -ary block-code generated by X , V_X , contains the code V as a subset.

1. Preliminaries

Definition 1.1. An algebra $(X, *, \theta)$ of type $(2, 0)$ is called a *BCI-algebra* if the following conditions are fulfilled:

- 1) $((x * y) * (x * z)) * (z * y) = \theta$, for all $x, y, z \in X$;
- 2) $(x * (x * y)) * y = \theta$, for all $x, y \in X$;
- 3) $x * x = \theta$, for all $x \in X$;
- 4) For all $x, y, z \in X$ such that $x * y = \theta, y * x = \theta$, it results $x = y$.

If a *BCI*-algebra X satisfies the following identity:

- 5) $\theta * x = \theta$, for all $x \in X$, then X is called a *BCK-algebra*.

A *BCK*-algebra X is called *commutative* if $x * (x * y) = y * (y * x)$, for all $x, y \in X$ and *implicative* if $x * (y * x) = x$, for all $x, y \in X$. A *BCK*-algebra $(A, *, \theta)$ is called *positive implicative* if and only if

$$(x * y) * z = (x * z) * (y * z), \text{ for all } x, y, z \in A.$$

The partial order relation " \leq " on a *BCK*-algebra is defined such that $x \leq y$ if and only if $x * y = \theta$.

An equivalent definition of *BCK*-algebra was gave in the following proposition.

Proposition 1.2. ([Me, Ju; 94], Theorem 1.6) *An algebra $(X, *, \theta)$ of type $(2, 0)$ is a BCK-algebra if and only if the following conditions are satisfied:*

- 1) $((x * y) * (x * z)) * (z * y) = \theta$, for all $x, y, z \in X$;
- 2) $x * (0 * y) = x$, for all $x, y \in X$;
- 3) For all $x, y, z \in X$ such that $x * y = \theta, y * x = \theta$, it results $x = y$.

Let $(X, *, \theta)$ be a finite *BCK*-algebra with n elements and A be a finite nonempty set. A map $f : A \rightarrow X$ is called a *BCK-function*. Let $A_n = \{0, 1, 2, \dots, n-1\}$. In the following, we will consider *BCK* algebra X and the set A under the form: $X = \{r_0, r_1, \dots, r_{n-1}\}, A = \{x_0, x_1, \dots, x_{m-1}\}, m \leq n$. A *cut function* of f is a map $f_{r_j} : A \rightarrow A_n, r_j \in X$, such that $f_{r_j}(x_i) = k$ if and only if $r_j * f(x_i) = r_k$, for all $r_j, r_k \in X, x_i \in A, i, j, k \in \{0, 1, 2, \dots, n-1\}$. For each *BCK*-function $f : A_n \rightarrow X$, we can define an n -ary block-code with codewords of length m . For this purpose, we consider to each element $r \in X$ the cut function $f_r : A \rightarrow A_n, r \in X$. To each such a function, will correspond the codeword w_r , with symbols from the set A_n . We have $w_r = w_0 w_1 \dots w_{n-1}$, with $w_i = j, j \in A_n$, if and only if $f_r(x_i) = j$, that means $r * f(i) = r_j$. We

denote this code with V_X . In this way, we can associate to each BCK -algebra an n -ary block code.

Example 1.3. We consider the following BCK -algebra $(X, *, \theta)$, with the multiplication given in the following table (see [Ju, So; 11], Example 4.2).

$*$	θ	a	b	c
θ	θ	θ	θ	θ
a	a	θ	θ	a
b	b	a	θ	b
c	c	c	c	θ

We have $X = \{\theta, a, b, c\}$, $A = A_4 = \{0, 1, 2, 3\}$. We consider $f : A \rightarrow X$, $f(0) = \theta$, $f(1) = a$, $f(2) = b$, $f(3) = c$ and $f_r : A_4 \rightarrow A_4$, $r \in X$, a cut function.

To $r = \theta$, corresponds the codeword $w_\theta = 0000$. For $r = a$, we obtain the codeword 1001. Indeed, $f_a(0) = 1$, since $a * f(0) = a * \theta = a = f(1)$; $f_a(a) = 0$ since $a * f(1) = a * a = \theta = f(0)$; $f_a(b) = 0$ and $a * f(2) = a * b = \theta = f(0)$; $f_a(c) = 1$, also $a * f(3) = a * c = a = f(1)$;

We wonder if and in what circumstances the converse is also true?
In the following, we will try to find answers at this question.

2. Main results

Let $A'_n = \{1, 2, \dots, n-1\}$ be a finite set and $V = \{w_1, w_2, \dots, w_m\}$ be n -ary codewords, ascending ordered after lexicographic order. We consider $w_i = w_{i1}w_{i2}\dots w_{iq}$, $w_{ij} \in A'_n$, $j \in \{1, 2, \dots, q\}$, with w_{ij} descending ordered such that

$$w_{iw_{ik}} \leq k, \quad i \in \{1, 2, \dots, m\}, \quad k \in \{1, 2, \dots, \min\{n-1, q\}\}$$

and $w_{ij} = 1$ in the rest.

Definition 2.1. Let V be the n -ary codeword, defined above. To this code we associate a matrix $M = (\alpha_{st})_{s,t \in \{0,1,\dots,r-1\}}$, $M \in \mathcal{M}_r(A_n)$, where r is defined in the following.

Case 1. $q < n$. Let $r = n-1+m$. We define $\alpha_{ss} = 0$, $\alpha_{s0} = s$, $\alpha_{0s} = 0$, $s \in \{0, 1, 2, \dots, r-1\}$. For $1 \leq s \leq n-1$, put $\alpha_{st} = 1$, if $t \leq s$, $\alpha_{st} = 0$, if $t \geq s$. For $s \geq n-1$, define $\alpha_{st} = w_{it}$, for $t \in \{1, 2, \dots, q\}$ and $\alpha_{sq+j} = 1$, for $q+j < s$. We have $\alpha_{st} = 0$, for $t \geq s$.

Case 2. $q \geq n$. Let $r = m+q+1$. We define $\alpha_{ss} = 0$, $\alpha_{s0} = s$, $\alpha_{0s} = 0$, $s \in \{0, 1, 2, \dots, r-1\}$. For $1 \leq s \leq q$, define $\alpha_{st} = 1$, if $t \leq s$, $\alpha_{st} = 0$, if $t \geq s$. For $s > q$, put $\alpha_{st} = w_{it}$, for $t \in \{1, 2, \dots, q\}$ and $\alpha_{sq+j} = 1$, for $q+j < s$. We have $\alpha_{st} = 0$, for $t \geq s$.

The matrix M is called *the matrix associated to the n -ary block code $V = \{w_1, w_2, \dots, w_m\}$* and is a lower triangular matrix. Example of such a matrix can be found in Section 3.

Definition 2.2. With the above notations, let $M \in \mathcal{M}_r(A_n)$ be the matrix associated to the n -ary block code $V = \{w_1, w_2, \dots, w_m\}$ defined on A'_n and $A_r = \{0, 1, \dots, r-1\}$ be a nonempty set. We define on A_r the following multiplication

$$i * j = \alpha_{ij} = w_{ij} = k.$$

Theorem 2.3. *With the above notations, we have that $(A_r, *, 0)$ is a BCK-algebra.*

Proof. Since conditions 2), 3) from Proposition 1.2 are satisfied using Definition 2.1, we will only prove that $((i * j) * (i * k)) * (k * j) = 0$, for all $i, j, k \in \{0, 1, \dots, r-1\}$.

Case 1: $j = 0, k \neq 0$. We will prove that $(i * (i * k)) * k = 0$. For $i = 0$ it is clear.

For $k = 0$, we obtain $(i * (i * 0)) * 0 = (i * i) * 0 = 0$.

For $k \neq 0, i \geq r - m, k \in \{1, 2, \dots, q\}$, we have $(i * (i * k)) = w_{i w_{ik}} \leq k$, therefore $(i * (i * k)) * k = 0$.

For $k \neq 0, i \geq r - m, k \geq q + 1, i \geq k$, we have $(i * (i * k)) * k = 0$, since $i * k = 1, i * 1 \leq n - 1 < k$.

For $i < r - m, k \leq q + 1$, we have $(i * (i * k)) * k = 0$ since $i * k = 1, i * 1 = 1$ and $1 * k = 0$.

For $i < r - m, k > q + 1$, we have $(i * (i * k)) * k = 0$ since $i * k = 0$, we obtain $(i * 0) * k = i * k = 0$.

Case 2: $k = 0, j \neq 0$. We will prove that $(i * j) * i = 0$. We always have that $i * j \leq i$, therefore $(i * j) * i = 0$.

Case 3: $k \neq 0, j \neq 0$. We will prove that $((i * j) * (i * k)) * (k * j) = 0$. For $i = 0$, it is clear. We suppose that $i \neq 0$.

For $i \geq r - m$ and $j, k < r - m, j < k$. We have $n - 1 \geq (i * j) \geq (i * k)$, therefore $((i * j) * (i * k)) = 1$. We also obtain $k * j = 1$, therefore $((i * j) * (i * k)) * (k * j) = 1 * 1 = 0$.

For $i \geq r - m$ and $j, k < r - m, k < j$. We have $n - 1 \geq (i * j) \leq (i * k)$, therefore $((i * j) * (i * k)) = 0$. It results that $((i * j) * (i * k)) * (k * j) = 0$.

For $i \geq r - m$ and $j, k \geq r - m, j < k$. We can have $i * j = 1$ and $i * k = 1$, therefore $(i * j) * (i * k) = 0$. We can also have $i * j = 1, i * k = 0$ and $k * j = 1$, since $j < k$. It results that $((i * j) * (i * k)) * (k * j) = (1 * 0) * 1 = 1 * 1 = 0$. Or, we can have $i * j = 0, i * k = 0$, therefore the asked relation is zero.

For $i \geq r - m$ and $j, k \geq r - m, k < j$. We can have $i * j = 1$ and $i * k = 1$, therefore $(i * j) * (i * k) = 0$. Or, we can have $i * k = 1, i * j = 0$ and $k * j = 0$, therefore we obtain zero. We also can have $i * j = 0, i * k = 0$, therefore the asked relation is zero.

For $i \geq r - m$ and $k < r - m < j$. We can have $i * j = 0$, therefore the asked relation is zero. We can have $i * j = 1$. It results $((i * j) * (i * k)) * (k * j) = (1 * (i * k)) * 0 = 1 * \beta = 0$, since $k < j$ and $\beta \geq 0$.

For $i \geq r - m$ and $j < r - m < k$. We have $i * j = 1$. If $i * k = 1$, we obtain zero. If $i * k = 0$, it results $((i * j) * (i * k)) * (k * j) = (1 * 0) * (k * j) = 1 * (k * j) = 0$, since $k * j \geq 1$.

For $i < r - m$ and $j, k < r - m, j < k$. We have $i * j = 1, i * k = 1$, therefore we obtain zero.

For $i < r - m$ and $j, k < r - m, k < j$. We can have $((i * j) * (i * k)) * (k * j) = (1 * 1) * 0 = 0$. Or, we can have $(i * j) = 0$, therefore we obtain zero.

For $i < r - m$ and $j, k < r - m, j < n - 1 + \max\{q, m\} - m \leq k$. We have $i * j = 1, i * k = 0$ and $k * j = 1$. It results $((i * j) * (i * k)) * (k * j) = (1 * 0) * 1 = 1 * 1 = 0$.

For $i < r - m$ and $k < r - m, k < r - m \leq j$. We can have $((i * j) * (i * k)) * (k * j) = (1 * 1) * 0 = 0$. Or, we can have $(i * j) = 0$, therefore we obtain zero.

For $i < r - m$ and $j, k \geq r - m, j < k$. We have $(i * j) = 0$, therefore we obtain zero.

For $i < r - m$ and $j, k \geq r - m, j > k$. We have $(i * j) = 0$, therefore we obtain zero. \square

Remark 2.4.

1) BCK -algebra $(A_r, *, 0)$ obtained in Theorem 2.3 is unique up to an isomorphism.

2) From Theorem 2.3, let $(A_r, *, 0)$ be the obtained BCK -algebra, with $A_r = \{0, 1, 2, \dots, r - 1\}$. If $X = \{a_0 = \theta, a_1, a_2, \dots, a_{r-1}\}$, with multiplication " \circ " given by the relation $a_i \circ a_j = a_k$ if and only if $i * j = k$, for $i, j, k \in \{0, 1, 2, \dots, r - 1\}$, then (X, \circ, θ) is a BCK -algebra.

3) If we consider $A_q = \{0, 1, 2, \dots, q - 1\}$, the map $f : A_q \rightarrow X, f(i) = a_i$, gives us a code V_X , associated to the above BCK -algebra (X, \circ, θ) , which contains the code V as a subset.

Definition 2.5. Let $(X, *, \theta)$ be a BCK -algebra, and $I \subseteq X$. We say that I is a *right-ideal* for the algebra X if $\theta \in I$ and $x \in I, y \in X$ imply $x * y \in I$. An ideal I of a BCK -algebra X is called a *closed ideal* if it is also a *subalgebra* of X (i.e. $\theta \in I$ and if $x, y \in I$ it results that $x * y \in I$).

Let V be an n -ary block code. From Theorem 2.3 and Remark 2.4, we can find a BCK -algebra X such that the obtained n -ary block-code V_X contains the n -ary block-code V as a subset.

Let V be a binary block code with m codewords of length q . With the above notations, let X be the associated BCK -algebra and $W = \{\theta, w_1, \dots, w_r\}$ the associated n -ary block code which include the code V . We consider the codewords $\theta, w_1, w_2, \dots, w_r$ lexicographically ordered, $\theta \geq_{lex} w_1 \geq_{lex} w_2 \geq_{lex} \dots \geq_{lex} w_r$. Let $M \in \mathcal{M}_r(A_n)$ be the associated matrix with the rows θ, w_1, \dots, w_r , in this order. Let L_{w_i} and C_{w_j} be the lines and columns in the matrix M . We consider the sub-matrix M' of the matrix M with the rows L_{w_1}, \dots, L_{w_m} and the columns $C_{w_{m+1}}, \dots, C_{w_{m+q}}$, which is the matrix associated to the code C .

Proposition 2.6. *With the above notations, we have that $\{\theta, w_1, w_{r-m}, w_{r-m+1}, \dots, w_r\}$ determines a closed right ideal in the algebra X .*

Proof. Let $Y = \{\theta, w_1, w_{r-m}, w_{r-m+1}, \dots, w_r\}$. We will prove that $y \in Y, x \in X$ imply $y * x \in Y$. From the definition of the multiplication in the algebra X , we have that $y * x \in \{\theta, w_1\}$. In the same time, if $x, y \in Y$, it results that $x * y \in Y$, since $y * x \in \{\theta, w_1\}$.

3. Examples

Example 3.1. Consider $A_7 = \{0, 1, 2, 3, 4, 5, 6\}$, $n = 7$, $q = 4$, $m = 3$, $r = 9$, $V = \{w_1, w_2, w_3\}$, with $w_1 = 3211, w_2 = 4221, w_3 = 4321$. The matrix M associated to the n -ary code V , is

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 6 & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{1} & 1 & 0 & 0 & 0 \\ 7 & \mathbf{4} & \mathbf{2} & \mathbf{2} & \mathbf{1} & 1 & 1 & 0 & 0 \\ 8 & \mathbf{4} & \mathbf{3} & \mathbf{2} & \mathbf{1} & 1 & 1 & 1 & 0 \end{pmatrix}$$

and the corresponded BCK -algebra, $(X, *, \theta)$, where

$X = \{a_0 = \theta, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$,

with the following multiplication table

*	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
θ	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_1	a_1	θ	θ	θ	θ	θ	θ	θ	θ
a_2	a_2	a_1	θ	θ	θ	θ	θ	θ	θ
a_3	a_3	a_1	a_1	θ	θ	θ	θ	θ	θ
a_4	a_4	a_1	a_1	a_1	θ	θ	θ	θ	θ
a_5	a_5	a_1	a_1	a_1	a_1	θ	θ	θ	θ
a_6	a_6	\mathbf{a}_3	\mathbf{a}_2	\mathbf{a}_1	\mathbf{a}_1	a_1	θ	θ	θ
a_7	a_7	\mathbf{a}_4	\mathbf{a}_2	\mathbf{a}_2	\mathbf{a}_1	a_1	a_1	θ	θ
a_8	a_8	\mathbf{a}_4	\mathbf{a}_3	\mathbf{a}_2	\mathbf{a}_1	a_1	a_1	a_1	θ

If we consider $A = \{1, 2, 3, 4\}$. The map $f : A \rightarrow X$, $f(1) = a_1, f(2) = a_2, f(3) = a_3, f(4) = a_4$ gives us the following block code

$V' = \{0000, 1000, 1100, 1110, 1111, \mathbf{3211}, \mathbf{4221}, \mathbf{4321}\}$, which contains V as a subset.

We remark that this algebra is not commutative since $a_7 * (a_7 * a_6) = a_7 * a_1 = a_4$ and $a_6 * (a_6 * a_7) = a_6 * \theta = a_6$. This algebra is not implicative since $a_6 * (a_7 * a_6) = a_6 * a_1 = a_3 \neq a_6$. This algebra is not positive implicative since $(x * y) * z \neq (x * z) * (y * z)$. Indeed, $(a_7 * a_6) * a_3 = a_1 * a_3 = \theta \neq (a_7 * a_3) * (a_6 * a_3) = a_2 * a_1 = a_1$.

Example 3.2. Let $A_4 = \{0, 1, 2, 3\}$, $n = 4$, $q = 5$, $m = 3$, $r = 9$, $V = \{w_1, w_2, w_3\}$, with $w_1 = 21111, w_2 = 32111, w_3 = 33111$. We obtain the matrix M associated to the n -ary code V ,

$$M = \begin{pmatrix} \begin{array}{cccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 6 & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 & 0 & 0 \\ 7 & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 0 & 0 & 0 \\ 8 & \mathbf{3} & \mathbf{3} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 1 & 1 & 0 \end{array} \end{pmatrix}$$

and the corresponded *BCK*-algebra, $(X, *, \theta)$, where

$X = \{a_0 = \theta, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$,

with the following multiplication table

$*$	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
θ	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_1	a_1	θ	θ	θ	θ	θ	θ	θ	θ
a_2	a_2	a_1	θ	θ	θ	θ	θ	θ	θ
a_3	a_3	a_1	a_1	θ	θ	θ	θ	θ	θ
a_4	a_4	a_1	a_1	a_1	θ	θ	θ	θ	θ
a_5	a_5	a_1	a_1	a_1	a_1	θ	θ	θ	θ
a_6	a_6	$\mathbf{a_2}$	$\mathbf{a_1}$	$\mathbf{a_1}$	$\mathbf{a_1}$	$\mathbf{a_1}$	θ	θ	θ
a_7	a_7	$\mathbf{a_3}$	$\mathbf{a_2}$	$\mathbf{a_2}$	$\mathbf{a_1}$	$\mathbf{a_1}$	a_1	θ	θ
a_8	a_8	$\mathbf{a_3}$	$\mathbf{a_3}$	$\mathbf{a_1}$	$\mathbf{a_1}$	$\mathbf{a_1}$	a_1	a_1	θ

If we consider $A = \{1, 2, 3, 4, 5\}$. The map $f : A \rightarrow X$, $f(1) = a_1, f(2) = a_2, f(3) = a_3, f(a_4) = 4, f(a_5) = 5$, gives us the following block code $V_X = \{00000, 10000, 11000, 11100, 11110, \mathbf{21111}, \mathbf{32211}, \mathbf{33111}\}$, which contains V as a subset.

Example 3.3. We consider $A_4 = \{0, 1, 2, 3\}$, $n = 4$, $q = 5$, $m = 5$, $r = 11$, $V = \{w_1, w_2, w_3, w_4, w_5\}$, with $w_1 = 11111, w_2 = 21111, w_3 = 31111, w_4 = 32111, w_5 = 33111$. We obtain the matrix M associated to the n -ary code V ,

$$M = \begin{pmatrix} \begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & \mathbf{3} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 1 & 0 & 0 & 0 & 0 \\ 9 & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 1 & 1 & 0 & 0 & 0 \\ 10 & \mathbf{3} & \mathbf{3} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 1 & 1 & 1 & 1 & 0 & 0 \end{array} \end{pmatrix}$$

and the corresponded *BCK*-algebra, $(X, *, \theta)$, where

$X = \{a_0 = \theta, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$,

with the following multiplication table

*	θ	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
θ	θ	θ	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_1	a_1	θ	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_2	a_2	a_1	θ	θ	θ	θ	θ	θ	θ	θ	θ
a_3	a_3	a_1	a_1	θ	θ	θ	θ	θ	θ	θ	θ
a_4	a_4	a_1	a_1	a_1	θ	θ	θ	θ	θ	θ	θ
a_5	a_5	a_1	a_1	a_1	a_1	θ	θ	θ	θ	θ	θ
a_6	a_6	a ₁	a ₁	a ₁	a ₁	a ₁	θ	θ	θ	θ	θ
a_7	a_7	a ₂	a ₁	a ₁	a ₁	a ₁	a_1	θ	θ	θ	θ
a_8	a_8	a ₃	a ₁	a ₁	a ₁	a ₁	a_1	a_1	θ	θ	θ
a_9	a_9	a ₃	a ₂	a ₁	a ₁	a ₁	a_1	a_1	a_1	θ	θ
a_{10}	a_{10}	a ₃	a ₃	a ₁	a ₁	a ₁	a_1	a_1	a_1	a_1	θ

If we consider $A = \{1, 2, 3, 4, 5\}$. The map $f : A \rightarrow X$, $f(1) = a_1, f(2) = a_2, f(3) = a_3, f(a_4) = 4, f(a_5) = 5$, gives us the following block code $V' = \{00000, 10000, 11000, 11100, 11110, \mathbf{11111}, \mathbf{21111}, \mathbf{31111}, \mathbf{32111}, \mathbf{33111}\}$, which contains V as a subset.

Conclusions. In this paper, we proved that to each n -ary block code V we can associate a BCK -algebra X such that the n -ary block-code generated by X, V_X , contains the code V as a subset. This algebra is unique up to an isomorphism and X is not commutative, not implicative and not positive implicative BCK -algebra.

As a further research will be very interesting to study properties of the above constructed codes and how these codes in connections with their associated BCK -algebras.

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